



# **$\alpha$ -Decay Half-Lives for Pb Isotopes Within Gamow-Like Model**

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**Abstract:** We studied alpha decay half-lives of Pb isotopes in the range  $178 \leq A \leq 220$ , within Gamow-like model (GLM) which is based on Gamow theory. The empirical formulae like Royer formula, Universal Decay Low (UDL), Viola-Seaborg formula (VSS), Semi-empirical formula for Poenaru (SemFIS) and Denisov & Khudenko formula (DEKH) are used to calculate alpha decay half-lives. The results are compared with experimental data, and also with other theoretical models, the results are in a good agreement was achieved with experimental data.

**Keywords:**  $\alpha$ -Decay, Gamow-Like Model, Semi-empirical Formula

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## **1. Introduction**

The study of alpha decay is one of the important modes to describe nuclear structure [1], it can be successfully described by quantum theory [2]. The alpha decay was described in 1928 [3] in base of quantum tunneling. A simple formula for  $\alpha$ -decay half-lives was derived using Wentzel-Kramers-Brillouin (WKB) approximation theory by A. Zdeb et al [4] for penetration of the Coulomb barrier with a square well for the nuclear part. GLM depended only one adjustable parameter, the radius constant, which is possible to reproduce with a good accuracy all existing data for decays of even-even nuclei. There are more model that was used to determine alpha decay half-lives such as generalized the liquid drop model (GLDM) which are used by Royer to describe  $\alpha$ -decay half-lives [5]. By considering Yukawa proximity potential (YPP) from ground-state to ground-state, Javadimanesh et.al was used to calculate  $\alpha$ -decay half-lives [6]. Preformed cluster model (PCM) was used by Kumar, to evaluate alpha decay half-lives for superheavy nuclei [7]. Double-folding with the standard M3Y nucleon-nucleon interaction (M3YDDIM) was used by Ren to calculate alpha decay half-lives [8]. New empirical formula was used to calculate alpha decay half-lives by Akrawy, which is modified the Royer formula [9]. Akrawy used (GLM) to calculate  $\alpha$ -decay half-lives for Lv isotopes [10]. In this study

we calculated the alpha decay half-lives for Pb isotopes using Gamow-Like Model (GLM), we compared the results with CPPM theoretical model [11] and with experimental data. Also we used some empirical formula to calculate alpha decay half-lives and to compare with GLD, like Royer formula [12], Universal Decay Low (UDL) [13], Viola-Seaborg formula (VSS) [14], Semi-empirical formula for Poenaru et. Al. (SemFIS) [15], and finally with Denisov & Khudenko formula (DEKH) [16].

## **2. Material Methods**

A. Zdeb et. al [4], developed and used a simple model to calculate alpha decay half-lives, this formula based on Gamow theory. this model formula is contain Coulomb potential adjusted parameter.

The penetration probability of tunneling alpha particle through barrier potential are calculated within Wentzel-Kramers-Brillouin (WKB) approximation using the following integral [4, 17]

$$P = \exp \left[ -\frac{2}{\hbar} \int_R^b \sqrt{2\mu(V(r) - Q)} dr \right] \quad (1)$$

where  $\mu$  is reduced mass,  $Q$  is energy released.  $a$  and  $b$  are tunneling point of integral which are given as

$$R = r_0 (A_1^{1/3} + A_2^{1/3}) \quad (2)$$

Where R is the spherical square well radius,  $A_1$  and  $A_2$  are atomic number of parent and daughter nuclei.  $r_0$  value has been taken from Ref. [5],

And

$$b = \frac{Z_1 Z_2 e^2}{Q} \quad (3)$$

where  $e^2$  is the electron charge [18],  $Z_1$  and  $Z_2$  are the atomic number of parent and daughter nuclei respectively.

The potential energy  $V(r)$  is given by the following conditions

$$V(r) = \begin{cases} -V_0 & 0 \leq r \leq R \\ \frac{Z_1 Z_2 e^2}{r} & r > R \end{cases} \quad (4)$$

Where  $V_0$  is the depth of the potential well which is equal to  $V_0 = 25A_1$  [19]. The  $\alpha$ -decay half-life can be evaluate by using [20]

$$T_{1/2} = \frac{\ln 2}{vP} \quad (4)$$

Where  $v$  is the assault frequency which is equal to 1020 s<sup>-1</sup> [21].

#### 1. Royer empirical formula (Royer)

Royer predicted new formula for alpha decay half-lives [12, 22-23] by fitting 373 nuclei which have transition from ground-state to ground state.

$$\log_{10}[T_{1/2}(s)] = a - bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q}} \quad (5)$$

where  $a$ ,  $b$  and  $c$  are coefficient are determined by least square fitting method,  $A$  and  $Z$  are mass number of and atomic number of parent nuclei respectively, and  $Q$  represented energy released during the reaction.

#### Universal decay law formula (UDL)

Qi et al, was predicted the universal decay law for cluster decay from microscopic mechanism process of the radioactive decay [13], the relation is found to be as [24,25]

$$\log_{10}[T_{1/2}(s)] = aZ_c Z_d \sqrt{\frac{A}{Q_c}} + b\sqrt{AZ_c Z_d (A_d^{1/3} + A_c^{1/3})} + c \quad (6)$$

$$= a\chi' + b\rho' + c \quad (7)$$

where  $A = \sqrt{\frac{A_d A_c}{(A_d + A_c)}}$ , and the  $a$ ,  $b$  and  $c$  are constants

which are used was determined by fitting to the experimental data of  $\alpha$ -decay and cluster [26].  $b\rho' + c$  is the effects term that include the cluster in the parent nucleus.

#### 2. The Viola-Seaborg semi-empirical formula (VSS)

From the Geiger-Nuttall law, the VSS formula was determined by Sobiczewski et al. [27], is given as

$$\log_{10}[T_{1/2}(s)] = \frac{(aZ + b)}{\sqrt{Q}} + cZ + d + h_{\log} \quad (8)$$

Here  $Z$  is the parent atomic number,  $Q$  is the energy released during the reaction in unit MeV, half-life in seconds, the coefficients  $a$ ,  $b$ ,  $c$ , and  $d$  are adaptable parameters and the parameter  $h_{\log}$  evidence the hindrance factor related with odd proton and odd neutron numbers [28], as given by Viola et al. [29]. More recent value was determined by Tiekuan Dong et. Al [14].

#### 3. Semi-empirical formula for Poenaru et. Al. (SemFIS)

Poenaru et al. modified semi-empirical formula for  $\alpha$ -decay half-life which is based on fission theory for  $\alpha$ -decay yield which is given as [15]

$$\log_{10}[T_{1/2}(s)] = 0.43429K_s\chi - 20.446 \quad (9)$$

where

$$K_s = 2.52956Z_d \left[ \frac{A_d}{AQ_\alpha} \right]^{1/2} \left[ \arccos \sqrt{r} - \sqrt{r(1-r)} \right] \quad (10)$$

and

$$r = 0.423Q_\alpha \frac{(1.5874 + A_d^{1/3})}{Z_d} \quad (11)$$

and the numerical coefficient  $\chi$ , close to the unity, is a second order polynomial

$$\chi = B_1 + B_2y + B_3z + B_4y^2 + B_5yz + B_6z^2 \quad (12)$$

The reduced variable  $y$  and  $z$ , expressed the distance from the closest magic-plus-one neutron and proton numbers  $N_i$  and  $Z_i$  is given as

$$z = \frac{(N - N_i)}{(N_{i+1} - N_i)}, \quad N_i < N \leq N_{i+1} \quad (13)$$

$$y = \frac{(Z - Z_i)}{(Z_{i+1} - Z_i)}, \quad Z_i < Z \leq Z_{i+1} \quad (14)$$

with  $N_i = \dots, 51, 83, 127, 185, 229, \dots$ , and  $Z_i = \dots, 29, 51, 83, 127, \dots$ ; Bi have been obtained by using a high quality selected data set of  $\alpha$ -decay.

#### 4. Denisov & Khudenko formula (DEKH)

From the Guy Royer empirical formula [23], Denisov & Khudenko [16, 30] developed empirical formula for  $\alpha$ -decay half-life between ground state to ground state  $\alpha$ -transition of parent and daughter nuclei, the formula is evaluated of  $\alpha$ -decay half-lives for even-even, even-odd, odd-even and odd-odd nuclei:

$$\log_{10}[T_{1/2}(s)] = -a - \frac{bA^{1/6}\sqrt{Z}}{\mu} + \frac{cZ}{\sqrt{Q_\alpha}} + \frac{d\sqrt{\ell(\ell+1)}}{QA^{-1/6}} - e((-1)^\ell - 1) \quad (15)$$

Here  $A$  and  $Z$  are the mass number and atomic number of parent nucleus, respectively,  $\ell$  is the orbital moment of emitted  $\alpha$  particle, and  $\mu = (A/(A-4))^{1/6}$ .  $Q$  is the reaction energy value. The  $\alpha$ -particle emission from nuclei obeys the spin-parity selection rule [31,32]

$$\ell_{\min} = \begin{cases} \Delta j & \text{for even } \Delta j \text{ and } \pi_p = \pi_d \\ \Delta j + 1 & \text{for odd } \Delta j \text{ and } \pi_p = \pi_d \\ \Delta j & \text{for odd } \Delta j \text{ and } \pi_p \neq \pi_d \\ \Delta j + 1 & \text{for even } \Delta j \text{ and } \pi_p \neq \pi_d \end{cases} \quad (16)$$

where  $\Delta j = |j_p - j_d|$ ,  $j_p, \pi_p, j_d, \pi_d$  are spin and parity values of the parent and daughter nuclei, respectively, and  $\ell = \ell_{\min}$ , the  $a, b, c, d$ , and  $e$  are free parameters have been evaluated by Denisov et al.[30].

### 3. Results and Discussion

The theoretical  $\alpha$ -decay half-lives of Pb isotopes, which have atomic number  $Z = 82$  within the range  $A = 178-220$  have been studied by using Gamow-Like Model. The  $Q$ -value was taken in Ref. [33] which is evaluated experimentally are listed in Table 1. The evaluation of  $\alpha$ -

decay half-lives within GLM, which are compared with experimental data [26, 33] are given in Table 1.

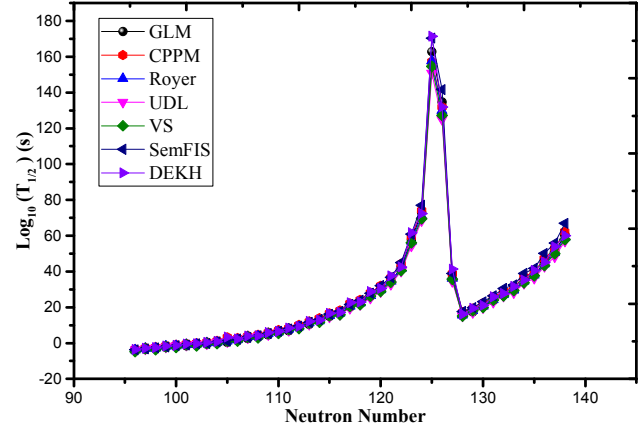


Figure 1. The comparison of the evaluated  $\alpha$ -decay half-lives of Pb isotopes.

The semi-empirical formulae was also been done using the analytical Royer formula (Royer), the universal decay law (UDL), the Viola-Seaborg formula (VSS), the Semi-empirical formula for Poenaru et. Al. (SemFIS)], and finally analytical Denisov & Khudenko formula (DEKH) have also been evaluated and presented in Table 1

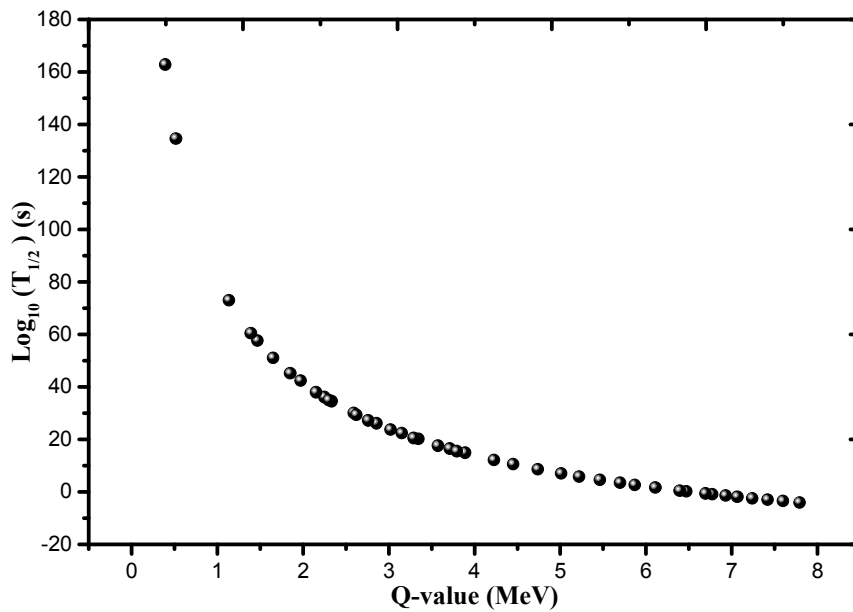
The maximum value of logarithm (T1/2) half-life is for the decay of the parent nuclei  $^{207}\text{Pb}$ , which have  $N=125$ , the maximum value of logarithm (T1/2) half-life which is reference the behavior nearly doubly magic number, this observation indicate the role of neutron shell close in  $\alpha$ -decay.

Table 1. The comparison of the evaluated  $\alpha$ -decay half-lives of Pb isotopes with experimental data and with other theoretical formula in the range  $178 \leq A \leq 220, Z = 82$ .

A	Q (MeV)	GLM	Exp.	CPPM	Royer	UDL	SemFIS	VSS	DEKH
178	7.790	-4.051	-3.638	-3.395	-3.809	-3.731	-4.724	-3.737	-3.616
179	7.598	-3.507	-2.409	-2.587	-2.877	-3.186	-3.083	-3.208	-2.790
180	7.419	-2.979	-2.377	-2.221	-2.705	-2.660	-3.593	-2.630	-2.496
181	7.240	-2.429	-1.409	-1.408	-1.766	-2.113	-1.951	-2.078	-1.595
182	7.066	-1.873	-1.260	-1.020	-1.573	-1.561	-2.436	-1.482	-1.348
183	6.928	-1.420	-0.272	-0.334	-0.730	-1.112	-0.894	-1.010	-0.481
184	6.774	-0.893	-0.310	0.037	-0.575	-0.592	-1.411	-0.453	-0.336
185	6.695	-0.625	0.799	0.520	0.081	-0.326	-0.057	-0.151	0.393
186	6.470	0.207	1.083	1.217	0.540	0.490	-0.271	0.705	0.796
187	6.393	0.490	2.185	3.217	1.214	0.769	1.096	1.036	3.356
188	6.109	1.638	2.185	2.750	1.984	1.889	1.191	2.207	2.261
189	5.870	2.671	2.431	4.037	3.421	2.892	3.299	3.310	3.984
190	5.697	3.456	3.591	4.679	3.810	3.654	3.028	4.111	4.112
191	5.460	4.604	4.250	5.938	5.364	4.763	5.243	5.347	5.809
192	5.221	5.846	5.902	7.210	6.195	5.958	5.414	6.610	6.532
193	5.010	7.019	6.551	8.410	7.776	7.083	7.645	7.886	8.377
194	4.738	8.657	-	10.152	8.983	8.649	8.195	9.556	9.359
195	4.450	10.563	-	12.097	11.295	10.464	11.130	11.594	12.122
196	4.226	12.181	-	13.818	12.456	11.999	11.648	13.252	12.882
197	3.889	14.896	-	16.606	15.565	14.566	15.351	16.131	16.666
198	3.709	16.492	-	18.243	16.679	16.072	15.838	17.782	17.165
199	3.343	20.164	-	22.164	20.725	19.520	20.441	21.655	22.590
200	3.151	22.340	-	24.201	22.374	21.559	21.472	23.925	22.940
201	2.857	26.111	-	28.182	26.517	25.082	26.150	27.915	28.827
202	2.590	30.090	-	32.017	29.879	28.790	28.884	32.079	30.552
203	2.335	34.525	-	36.609	34.674	32.911	34.172	36.761	37.617
204	1.969	42.375	-	44.243	41.711	40.183	40.545	44.988	42.552
205	1.468	57.638	-	59.408	56.947	54.271	56.008	60.877	61.670
206	1.135	73.027	-	74.146	71.068	68.436	69.415	77.063	72.323

Table 1. Continues.

A	Q (MeV)	GLM	Exp.	CPPM	Royer	UDL	SemFIS	VSS	DEKH
207	0.392	162.866	-	157.279	157.665	150.835	154.608	170.385	171.485
208	0.517	134.658	-	131.906	129.841	124.994	127.172	141.686	131.926
209	2.248	36.103	-	38.833	36.189	34.398	35.775	39.007	41.353
210	3.792	15.498	-	17.262	15.684	15.181	15.108	17.692	16.158
211	3.571	17.562	-	19.382	18.163	17.126	18.174	19.816	19.455
212	3.290	20.497	-	22.283	20.559	19.883	19.935	23.135	21.102
213	3.020	23.707	-	25.720	24.161	22.888	24.082	26.419	25.836
214	2.760	27.245	-	29.140	27.102	26.190	26.401	30.482	27.738
215	2.620	29.361	-	31.413	29.653	28.164	29.494	32.550	31.678
216	2.300	34.947	-	37.021	34.536	33.356	33.740	38.919	35.277
217	2.150	37.983	-	40.057	37.993	36.173	37.695	41.866	40.549
218	1.850	45.168	-	47.201	44.361	42.824	43.428	50.135	45.241
219	1.650	51.020	-	52.990	50.553	48.230	50.023	55.935	53.905
220	1.390	60.469	-	62.276	59.017	56.944	57.860	66.924	60.105

Figure 2. Correlation between  $Q$ -value vs. logarithm ( $T_{1/2}$ ) half-lives of Pb isotopes.

From Table 1, it can be seen clearly the GLM, CPPM, Royer, UDL, VSS, SemFIS and DEKH are the same orientation. It should be taken into account that GLM values match close to experimental data and Coulomb proximity potential model.

The correlation between  $Q$ -value and logarithm ( $T_{1/2}$ )  $\alpha$ -decay half-lives was shown in Figure 2, it show the logarithm ( $T_{1/2}$ )  $\alpha$ -decay half-lives exponential decrease with increasing  $Q$ -value

#### 4. Conclusion

The  $\alpha$ -decay half-lives for Pb isotopes within GLM have been performed. The comparison between GLM and experimental data have been done, also the our result was compared with CPPM and some empirical formulae such as Royer, UDL, VSS, SemFIS and DEKH, the result is match close to each other. The  $Q$ -value have the main role, which is decrease exponential with increasing logarithm ( $T_{1/2}$ ) half-life. From the result we predict by GLM, the maximum value of logarithm ( $T_{1/2}$ ) half-life of Pb isotopes is founded in  $N =$

125, which is influence of the neutron shell close.

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